PHY 103

Laminar Flow
Turbulent Flow
Reynolds number
Viscosity

The most important properties to consider when learning mechanics of fluid are:

① Density
② Viscosity
③ Surface tension

As already pointed out, different liquids flow at different rates given all other conditions remains same. This means there is some property that affects the fluids flow. This property is called viscosity. Viscosity of a fluid originates from the nature of molecular interactions.
Liquids, unlike gasses, have restricted molecular motion more or less a vibration with smaller amplitude than that for gas but higher than that of solids. When ~~liquid~~ liquids flows under applied shear, molecules are in motion and continuously dislocating from its molecular arrangement with respect to other molecules. To dislocate a molecule, a certain amount of energy is required.

Viscosity is the energy that needs to dislocate a mole of a fluid.

$$\eta = \frac{Nh}{\tilde{V}} e^{3.8 T_b / T}$$

Where η is the viscosity, N is the Avogadro Constant h is plank's constant, $\tilde{V}$ is volume of a mole of liquid, T_b is the boiling point and T is the temperature.

In general, viscosity of a fluid is a measure of its resistance to gradual deformation by shear stress or tensile stress. For liquids, it corresponds to the informal concept of thickness for example, honey has a much higher viscosity than water. In fluid mechanics, the Reynolds number (Re) is a dimensionless number that gives a measure of the ratio of inertial forces to viscous forces and ~~conse~~ consequently quantifies the relative importance of these two types of forces for given flow conditions.

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PHY 103

Laminar flow, Turbulent flow, Reynold's number and Viscosity

Laminar Flow

In fluid dynamics, Laminar flow (or streamline flow) occurs when a fluid flows in parallel layers, with no disruption between the layers. At low Velocity the fluid tends to flow without lateral mixing, and adjacent layers slides past one another like playing cards.

Laminar flow occur
① when the liquid viscosity is high
② liquid velocity is below a certain value
③ when the characteristic length is significant small

In laminar flow, the fluid particle does not experience any acceleration. Therefore the resultant ~~has to be~~ is zero. This force acting on the fluid element fact can be used to derived the velocity distribution of the fluid.

$$\rightarrow \quad \begin{matrix} d & \overset{A}{\diagup} & c \\ a & & b \end{matrix} \quad \leftarrow$$

Given that ~~Assume~~ P and τ are pressure and the shear stress at the centre A of the element. The length and the width of the element are dx and dy respectively.

If the pressure at the centre of the element is p, pressure at ad can be written as $p - \frac{dp}{dx}\frac{\partial x}{2}$ and the pressure at bc is $p - \frac{dp}{dx}\frac{\partial x}{2}$

∴ The total force exerted by pressure in x direction is:

$$F_p = \left(p - \frac{dp}{dx}\frac{\partial x}{2}\right)\partial x - \left(p + \frac{dp}{dx}\frac{\partial x}{2}\right)\partial y = -\frac{dp}{dx}\partial x \partial y$$

Similarly the total pressure exerted by the shear stress is

$$F_\tau = \left(\tau + \frac{d\tau}{dy}\frac{\partial y}{2}\right)\partial x - \left(\tau - \frac{d\tau}{dy}\frac{\partial y}{2}\right)\partial x = \frac{d\tau}{dy}\partial x \partial y$$

The weight of the fluid element in the flow direction

$$f_g = w\sin\theta = \rho g\, \partial x\, \partial y \sin\theta$$

Since the flow is steady

$$f_p + f_\tau + f_g = 0$$

$$\Rightarrow -\frac{dp}{dx} + \frac{d\tau}{dy} + \rho g \sin\theta = 0$$

If $-\frac{dz}{dx} = \sin\theta$

$$\therefore \frac{d\tau}{dy} = \frac{d}{dx}(p + \rho g z)$$

~~given that~~ from the definition, shear stress in a fluid is proportional to the rate of change of velocity across the ~~liquid~~ fluid path

$$\tau \propto \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy}$$

$$\therefore \mu \frac{d^2u}{dy^2} = \frac{d}{dx}(p + \rho g z)$$

Integrating the above equation ~~two time~~ twice w.r.t y

We have
~~The~~

$$\mu u = \left[\frac{d}{dx}(p + \rho g \xi)\right]\frac{y^2}{2} + C_1 y + C_2$$

where C_1 and C_2 are arbitrary constants arising from the indefinite integration. The fluid that flows is bounded by two boundaries.

Turbulent Flow

Turbulent flow, is a type of fluid flow ~~on~~ in which the fluid undergoes irregular fluctuation or mixing. In ~~the~~ ~~speed of the~~ turbulent flow, the speed of the fluid at a point is continuously undergoing changes in both magnitude and direction. Most flows occurring in nature and industrial applications are said to be turbulent.

It should be noted that even though the velocity fluctuations in turbulent flows, the mean velocity remains constant.

The parameter use ~~to~~ in fluid mechanics to classify flow regimes is Reynolds number (Re). Reynolds no. is defined as the ratio of inertial forces to the viscous forces

We can expect a fluid to be turbulent when V is very small

$$= Re = \frac{\rho U L}{\mu} = \frac{UL}{V} \quad \mu = \text{viscosity}$$

where $V = \frac{\rho}{\mu}$ is the kinematic viscosity
ρ = density, U = mean velocity
L = length (diameter)

~~Laminar flow Re 2300~~

Transition flow ~~2300 < Re~~ < 4000

Turbulence in fluid flow can be thought of a velocity fluctuations (u') around mean velocity.

~~$\therefore u = \bar{u} + u'$~~

Therefore the velocity can be written as

$$u = \bar{u} + u' \quad \bar{u} = \text{mean velocity}, \quad u' = \text{velocity fluctuation}$$

The mean kinetic energy E of the flow is given by

$$E = \frac{1}{2}\rho\bar{u}^2 = \frac{1}{2}\rho\,\overline{(\bar{u}+u')^2}$$

$$= \frac{1}{2}\rho\left[\overline{\bar{u}^2} + \overline{2\bar{u}u'} + \overline{u'}\right]$$

The overbar is used to indicate the mean value of the variable

$$\overline{(\bar{u})^2} = (\bar{u})^2, \quad \overline{u'} = 0$$

$$\Rightarrow E = \frac{1}{2}\rho\left[(\bar{u})^2 + \overline{(u')^2}\right]$$

~~where~~ $\frac{1}{2}$ $E = \frac{1}{2}\rho\overline{(u')^2} + \frac{1}{2}\rho(\bar{u})^2$

the kinetic energy of a turbulent flow consist of ~~two~~ part ~~turbulence~~

$\frac{1}{2}\rho(\bar{u}')^2$ = Kinetic energy of the mean velocity

$\frac{1}{2}\rho\overline{(u')^2}$ = Kinetic energy of the turbulence

Reynolds number for

The Value of laminar flow is less than 2300

i.e $Re < 2300$

the transitional flow for Re is

$2300 < Re < 5000$

Turbulent flow for Re is greater than 5000

i.e $Re > 5000$.

Example

Consider the fluid density (ρ) of 1000 kg/m³ passing through a pipe of diameter 0.5m with the viscosity of 0.55×10^{-3} Ns/m² with the Reynolds number of 3500. Calculate the maximum velocity of the fluid

Sol Recall

$$Re = \frac{\rho u L}{\mu}$$

$$3500 = \frac{1000 \times 0.5\, u}{0.55 \times 10^{-3}}$$

$$u = \frac{3500 \times 0.55 \times 10^{-3}}{1000 \times 0.5}$$

The Continuity Equation

The mass of a moving fluid does not change as it flows. This leads to an important quantitative relationship called continuity equation. Consider a portion of a flow tube between two stationary cross sections with areas A_1 and A_2. The fluid speeds at these sections are v_1 & v_2 respectively. No fluid flows in or out across the sides of the tube because the fluid velocity is tangent to the wall at every point on the wall.

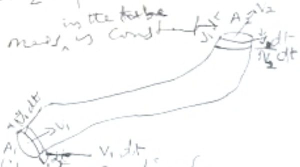
During a small time interval dt, the fluid at A_1 moves a distance $v_1 dt$, so a cylinder of fluid with length $v_1 dt$ and volume $dV_1 = A_1 v_1 dt$ flows into the tube across A_1. During this same interval, a cylinder of volume $dV_2 = A_2 v_2 dt$ flows out of the tube across A_2.

for incompressible fluid ~~(assumption)~~ with constant density.

The inflow mass at A_1 is $dm_1 = \rho A_1 v_1 dt$

the outflow mass at A_2 is $dm_2 = \rho A_2 v_2 dt$

In the steady flow the total mass in the tube is constant



$\Rightarrow dm_1 = dm_2$

$\rho A_1 v_1 dt = \rho A_2 v_2 dt$

so $A_1 v_1 = A_2 v_2$ — Continuity equation for incompressible fluid

The product Av is the volume flow rate $\frac{dV}{dt}$, the rate at which volume crosses a section of the tube.

$\Rightarrow \frac{dV}{dt} = AV$ — Volume flow rate.

The mass flow rate is the mass flow per unit time through a cross section. This is equal to the density ρ times the ~~rate~~ volume flow rate $\frac{dV}{dt}$.

From equation * if the density of the fluid is ~~not~~ not uniform (not constant at any point in the tube). Then equation * becomes continuity equation

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \quad \text{— for compressible fluid.}$$

If the fluid is denser at point 2 that at point 1 $(\rho_2 > \rho_1)$ The ~~V~~olume flow rate at point 2 will be less than at point 1.

Example: Incompressible oil of density 850 kg/m³ is pumped through a cylinder pipe at a rate of 9.5 liters per second.
(a) The first section of the pipe has a diameter of 8 cm. What is the flow speed of the oil? what is the mass flow rate?
(b) The second section of the ~~fluid~~ pipe has a diameter of 4 cm. What are the flow speed and mass flow rate in that section?

(a) (i) $\frac{dV}{dt} = A_1 v_1 \; ; \; v_1 = \frac{dV}{dt} \Big/ A_1 = \frac{9.5 \times 10^{-3}}{\pi (4 \times 10^{-2})^2} = 1.9\ \text{m/s}$

$A = \pi r^2 \qquad v_1 = 1.9\ \text{m/s}$

(ii) The mass flow rate is $\rho \frac{dV}{dt} = 850 \times 9.5 \times 10^{-3} = 8.1\ \text{kg/s}$

(b) from the continuity equation

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{\cancel{\pi} r_1^2 \times v_1}{\cancel{\pi} r_2^2} = \frac{(4 \times 10^{-2})^2}{(2 \times 10^{-2})^2} \times 1.9$$

$$v_2 = 7.6\ \text{m/s}$$

$$\Rightarrow v_2 = 4 v_1$$

Since the density is the same, the volume and mass flow rates are the same as in part a above.

Bernoulli's Equation

According to the continuity equation, the speed of fluid flow can vary along the paths of the fluid. The pressure can also vary; it depends on height as in the static situation and it also depends on the speed of flow.

We can derive an important relationship called Bernoulli's equation that relates the pressure, flow speed and height for flow of an ideal incompressible fluid.

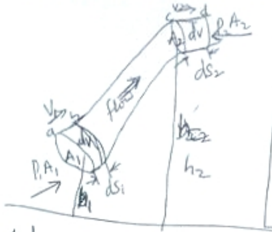
Bernoulli's equation is an essential tools in analysing plumbing system, hydroelectric generating station and the flight of airplanes.

The dependence of pressure on speed follows from the continuity equation, When an incompressible fluid flows along a flow tube with varying cross section, its speed must change, and so an element of fluid mass have an acceleration. If the tube is horizontal, the force that causes this acceleration has to be applied by the surrounding fluid. This mean that the pressure must be different in regions of different cross section: if it were the same everywhere, the net force on every fluid element would be zero. When a horizontal flow tube narrows and a fluid element speeds up, it must be moving toward a region of lower pressure in order to have a net forward to accelerate it. If the elevation changes this causes an additional pressure difference

To derive Bernoulli's equation, we apply work-energy theorem to the fluid in a section of a flow tube.

$$dW = P_1 A_1 ds_1 - P_2 A_1 ds_2 = (P_1 - P_2) dV \quad (1)$$

The work dW is due to forces other than the conservative force of gravity. So it equals the change



Caution: Bernoulli's eqn is valid for only incompressible steady flow of a fluid with no internal friction.

in the total mechanical energy (Kinetic plus gravitational potential energy)

Kinetic energy at $A_1 = \frac{1}{2}\rho(A_1 ds_1)V_1^2$

at $A_2 = \frac{1}{2}\rho(A_2 ds_2)V_2^2$

The net kinetic energy $dK = \frac{1}{2}\rho dv(V_2^2 - V_1^2)$ --- (2)

Also change in gravitational potential energy dU

at A_1 $dmgy_1 = \rho dvgh_1$

at A_2 $dmgy_2 = \rho dvgh_2$

$$dU = \rho dvg(h_2 - h_1) \quad \text{--- (3)}$$

Combining eqns 1, 2 and 3

$$(P_1 - P_2)dV = \frac{1}{2}\rho dV(V_2^2 - V_1^2) + \rho dVg(h_2 - h_1)$$

divide through by dV

$$P_1 - P_2 = \frac{1}{2}\rho(V_2^2 - V_1^2) + \rho g(h_2 - h_1) \quad \text{--- (4)}$$

Equation 4 is known as Bernoulli's Equation. It states that the work done on a unit volume of fluid by the surrounding fluid is equal to the sum of the changes in kinetic and potential energy per unit volume that occur during the flow. In terms of pressure, equation 4 becomes

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho gh_2$$

$\Rightarrow P + \rho gh + \frac{1}{2}\rho v^2$ = constant at any point along the flow ...